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PREFACE

The main purpose of this book is the development of a new method for the semantical analysis of meaning, that is, a new method for analyzing and describing the meanings of linguistic expressions. This method, called *the method of extension and intension*, is developed by modifying and extending certain customary concepts, especially those of class and property. The method will be contrasted with various other semantical methods used in traditional philosophy or by contemporary authors. These other methods have one characteristic in common: They all regard an expression in a language as a name of a concrete or abstract entity. In contradistinction, the method here proposed takes an expression, not as naming anything, but as possessing an intension and an extension.

This book may be regarded as a third volume of the series which I have called "Studies in Semantics", two volumes of which were published earlier. However, the present book does not presuppose the knowledge of its predecessors but is independent. The semantical terms used in the present volume are fully explained in the text. The present method for defining the L-terms (for example, 'L-true', meaning 'logically true', 'analytic') differs from the methods discussed in the earlier *Introduction to Semantics*. I now think that the method used in this volume is more satisfactory for languages of a relatively simple structure.

After meaning analysis, the second main topic discussed in this book is *modal logic*, that is, the theory of modalities, such as necessity, contingency, possibility, impossibility, etc. Various systems of modal logic have been proposed by various authors. It seems to me, however, that it is not possible to construct a satisfactory system before the meanings of the modalities are sufficiently clarified. I further believe that this clarification can best be achieved by correlating each of the modal concepts with a corresponding semantical concept (for example, necessity with L-truth). It will be seen that this method also leads to a clarification and elimination of certain puzzles which logicians have encountered in connection with modalities. In the Preface to the second volume of "Studies in Semantics," I announced my intention to publish, as the next volume, a book on modal logic containing, among other things, syntactical and semantical systems which combine modalities with quantification. The present book, however, is not as yet the complete fulfilment of that promise: it contains

12-2. ***Second Principle of Interchangeability.*** Let $..\mathfrak{A}_j..$ be a sentence (in S) which is either extensional or intensional with respect to a certain occurrence of the designator $\mathfrak{A}_j$, and $..\mathfrak{A}_k..$ the corresponding sentence with $\mathfrak{A}_k$.

a. If $\mathfrak{A}_j$ and $\mathfrak{A}_k$ are L-equivalent (in S), then the occurrence in question of $\mathfrak{A}_j$ within $..\mathfrak{A}_j..$ is L-interchangeable and hence interchangeable with $\mathfrak{A}_k$ (in S).

Formulations b and c of the second principle analogous to 12-1b and c are possible only with the help of a modal sign, hence only with respect to a nonextensional language system. They will be given later (39-7b and c).

The following theorems follow from the two principles just stated, with the help of the definitions of extensional and intensional systems (11-2c and 11-3c):

12-3. Let S be an *extensional system* (for instance, S_1, see Example IV in § 11).

a. Equivalent expressions are interchangeable in S.

b. L-equivalent expressions are L-interchangeable in S.

Examples. **a.** Equivalence and therefore interchangeability in S_1 hold for the following pairs of expressions: (i) 'H' and 'F•B' (see 3-8); (ii) 'Hs' and '(F•B)(s)'; (iii) 's' and '$(\iota x)(Axw)$' (see 9-2). **b.** L-equivalence and therefore L-interchangeability in S_1 hold for the following pairs of expressions: (i) 'H' and 'RA' (see 3-11); (ii) 'Hs' and 'RAs'; (iii) '$(\iota x)(Hx \bullet Axw)$' and '$(\iota x)(RAx \bullet Axw)$' (see § 9).

12-4. Let S be an *intensional system* (for instance, S_2 with the modal sign 'N', see Example II in § 11 and § 39).

a. Equivalent expressions are interchangeable in S, except where they occur in an intensional context (for example, in the system S_2: except in a context of the form 'N(. . .)').

b. L-equivalent expressions are L-interchangeable in S.

Examples for S_2. **a.** Let 'C' be F-true, as in Example II, § 11. Then 'C' and 'C V ~ C' are equivalent (see 11-4). The sentence '(C V ~ C)• N(C V ~ C)' is true (see 11-5). Within this sentence the first occurrence of 'C V ~ C' is interchangeable with 'C', while the second is not. **b.** For the pairs of L-equivalent expressions in S_1 mentioned above, L-equivalence in S_2 and therefore L-interchangeability in S_2 likewise hold.

§ 13. Sentences about Beliefs

We study sentences of the form 'John believes that . . .'. If here the subsentence '. . .' is replaced by another sentence L-equivalent to it, then it may be that the whole sentence changes its truth-value. Therefore, the whole belief-sentence is neither extensional nor intensional with respect to the subsentence '. . .'. Consequently, an interpretation of belief-sentences as referring either to sentences or to propositions is not quite satisfactory. For a more adequate interpretation we need a relation between sentences which is still stronger than L-equivalence. Such a relation will be defined in the next section.

We found that '. . . V - - -' is extensional with respect to the subsentence indicated by dots, and that 'N(. . .)' is intensional. Can there be a context which is neither extensional nor intensional? This would be the case if (but not only if) the replacement of a subsentence by an L-equivalent one changed the truth-value and hence also the intension of the whole sentence. In our systems this cannot occur; every sentence in S_1 (and likewise in S_3, to be explained later) is extensional, and every sentence in S_2 is either extensional or intensional. However, it is the case for a very important kind of sentence with psychological terms, like 'I believe that it will rain'. Although sentences of this kind seem to be quite clear and unproblematic at first glance and are, indeed, used and understood in everyday life without any difficulty, they have proved very puzzling to logicians who have tried to analyze them. Let us see whether we can throw some light upon them with the help of our semantical concepts.

In order to formulate examples, we take here, as our object language S, not a symbolic system but a part of the English language. We assume that S is similar in structure to S_1 except for containing the predicator '. . believes that - -' and some mathematical terms. We do not specify here the rules of S; we assume that the semantical rules of S are such that the predicator mentioned has its ordinary meaning; and, further, that our semantical concepts, especially 'true', 'L-true', 'equivalent', and 'L-equivalent', are defined for S in accord with our earlier conventions. Now we consider the following two belief-sentences; 'D' and 'D′' are here written as abbreviations for two sentences in S to be explained presently:

(i) 'John believes that D'.
(ii) 'John believes that D′'.

Suppose we examine John with the help of a comprehensive list of sentences which are L-true in S; among them, for instance, are translations into English of theorems in the system of [P.M.] and of even more complicated mathematical theorems which can be proved in that system and therefore are L-true on the basis of the accepted interpretation. We ask

John, for every sentence or for its negation, whether he believes what it says or not. Since we know him to be truthful, we take his affirmative or negative answer as evidence for his belief or nonbelief. Among the simple L-true sentences, there will certainly be some for which John professes belief. We take as 'D' any one of them, say 'Scott is either human or not human'. Thus the sentence (i) is true. On the other hand, since John is a creature with limited abilities, we shall find some L-true sentences in S for which John cannot profess belief. This does not necessarily mean that he commits the error of believing their negations; it may be that he cannot give an answer either way. We take as 'D″' some sentence of this kind; that is to say, 'D″' is L-true but (ii) is false. Thus the two belief-sentences (i) and (ii) have different truth-values; they are neither equivalent nor L-equivalent. Therefore, the definitions of interchangeability and L-interchangeability (11-1a) lead to the following two results:

13-1. The occurrence of 'D' within (i) is not interchangeable with 'D″'.
13-2. The occurrence of 'D' within (i) is not L-interchangeable with 'D″'.

'D' and 'D″' are both L-true; therefore:

13-3. 'D' and 'D″' are equivalent and L-equivalent.

Examining the first belief-sentence (i) with respect to its subsentence 'D', we see from 13-1 and 13-3 that the condition of extensionality (11-2a) is not fulfilled; and we see from 13-2 and 13-3 that the condition of intensionality (11-3a) is not fulfilled either:

13-4. The belief-sentence (i) is *neither extensional nor intensional* with respect to its subsentence 'D'.

Although 'D' and 'D″' have the same intension, namely, the L-true or necessary proposition, and hence the same extension, namely, the truth-value truth, their interchange transforms the first belief-sentence (i) into the second (ii), which does not have the same extension, let alone the same intension, as the first.

The same result as 13-4 holds also if any other sentence is taken instead of 'D', in particular, any factual sentence.

Let us now try to answer the much-discussed question as to how a sentence reporting a belief is to be analyzed and, in particular, whether such a sentence is about a proposition or a sentence or something else. It seems to me that we may say, in a certain sense, that (i) is about the sentence 'D', but also, in a certain other sense, that (i) is about the proposition that D. In interpreting (i) with respect to the sentence 'D', it would, of course, not do to transform it into 'John is disposed to an affirmative response to the sentence 'D' ', because this might be false, although (i) was assumed to be true; it might, for instance, be that John does not understand English but expresses his belief in another language. Therefore, we may try the following more cautious formulation:

(iii) 'John is disposed to an affirmative response to some sentence in some language, which is L-equivalent to 'D' '.

Analogously, in interpreting (i) with respect to the proposition that D, the formulation 'John is disposed to an affirmative response to any sentence expressing the proposition that D' would be wrong because it implies that John understands all languages. Even if the statement is restricted to sentences of the language or languages which John understands, it would still be wrong, because 'D″', for example, or any translation of it, likewise expresses the proposition that D, but John does not give an affirmative response to it. Thus we see that here again we have to use a more cautious formulation similar to (iii):

(iv) 'John is disposed to an affirmative response to some sentence in some language which expresses the proposition that D'.

However, it seems to me that even the formulations (iii) and (iv), which are L-equivalent, should not be regarded as anything more than a first approximation to a correct interpretation of the belief-sentence (i). It is true that each of them follows from (i), at least if we take 'belief' here in the sense of 'expressible belief', leaving aside the problem of belief in a wider sense, interesting though it may be. However, (i) does not follow from either of them. This is easily seen if we replace 'D' by 'D″'. Then (iii) remains true because of 13-3; on the other hand, (i) becomes (ii), which is false. It is clear that we must interpret (i) as saying as much as (iii) but something more; and this additional content seems difficult to formulate. If (i) is correctly interpreted in accord with its customary meaning, then it follows from (i) that there is a sentence to which John would respond affirmatively and which is not only L-equivalent to 'D', as (iii) says, but has a still stronger relation to 'D'—in other words, a sentence which has something more in common with 'D' than the intension. The two sentences must, so to speak, be understood in the same way; they must not only be L-equivalent in the whole but consist of L-equivalent parts, and both must be built up out of these parts in the same way. If this is the case, we shall say that the two sentences have the same intensional structure. This concept will be explicated in the next section and applied in the analysis of belief sentences in § 15.

§ 14. Intensional Structure

If two sentences are built in the same way out of designators (or designator matrices) such that any two corresponding designators are L-equivalent, then we say that the two sentences are *intensionally isomorphic* or that they have the same *intensional structure*. The concept of L-equivalence can also be used in a wider sense for designators in different language systems; and the concept of intensional isomorphism can then be similarly extended.

We shall discuss here what we call the analysis of the intensional structures of designators, especially sentences. This is meant as a semantical analysis, made on the basis of the semantical rules and aimed at showing, say for a given sentence, in which way it is built up out of designators and what are the intensions of these designators. If two sentences are built in the same way out of corresponding designators with the same intensions, then we shall say that they have the same intensional structure. We might perhaps also use for this relation the term 'synonymous', because it is used in a similar sense by other authors (e.g., Langford, Quine, and Lewis), as we shall see in the next section. We shall now try to explicate this concept.

Let us consider, as an example, the expressions '2 + 5' and 'II sum V' in a language S containing numerical expressions and arithmetical functors. Let us suppose that we see from the semantical rules of S that both '+' and 'sum' are functors for the function Sum and hence are L-equivalent; and, further, that the numerical signs occurring have their ordinary meanings and hence '2' and 'II' are L-equivalent to one another, and likewise '5' and 'V'. Then we shall say that the two expressions are *intensionally isomorphic* or that they have *the same intensional structure*, because they not only are L-equivalent as a whole, both being L-equivalent to '7', but consist of three parts in such a way that corresponding parts are L-equivalent to one another and hence have the same intension. Now it seems advisable to apply the concept of intensional isomorphism in a somewhat wider sense so that it also holds between expressions like '2 + 5' and 'sum(II,V)', because the use in the second expression of a functor preceding the two argument signs instead of one standing between them or of parentheses and a comma may be regarded as an inessential syntactical device. Analogously, if '>' and 'Gr' are L-equivalent, and likewise '3' and 'III', then we regard '5 > 3' as intensionally isomorphic to 'Gr(V,III)'. Here again we regard the two predicators '>' and 'Gr' as corresponding to each other, irrespective of their places in the sentences; further, we correlate the first argument expression of '>' with the first of 'Gr', and the second with the second. Further, '2 + 5 > 3' is isomorphic to 'Gr[sum(II,V),III]', because the corresponding expressions '2 + 5' and 'sum(II,V)' are not only L-equivalent but isomorphic. On the other hand, '7 > 3' and 'Gr[sum(II,V),III]' are not isomorphic; it is true that here again the two predicators '>' and 'Gr' are L-equivalent and that corresponding argument expressions of them are likewise L-equivalent, but the corresponding expressions '7' and 'sum(II,V) are not isomorphic. We require for isomorphism of two expressions that the analysis of both down to the smallest subdesignators lead to analogous results.

We have said earlier (§ 1) that it seems convenient to take as designators in a system S at least all those expressions in S, but not necessarily only those, for which there are corresponding variables in the metalanguage M. For the present purpose, the comparison of intensional structures, it seems advisable to go as far as possible and take as designators all those expressions which serve as sentences, predicators, functors, or individual expressions of any type, irrespective of the question of whether or not M contains corresponding variables. Thus, for example, we certainly want to regard as isomorphic '$p \vee q$' and 'Apq', where 'A' is the sign of disjunction (or alternation) as used by the Polish logicians in their parenthesis-free notation, even if M, as is usual, does not contain variables of the type of connectives. We shall then regard '$\vee$' and 'A' as L-equivalent connectives because any two full sentences of them with the same argument expressions are L-equivalent.

Frequently, we want to compare the intensional structures of two expressions which belong to different language systems. This is easily possible if the concept of L-equivalence is defined for the expressions of both languages in such a way that the following requirement is fulfilled, in analogy to our earlier conventions: an expression in S is L-equivalent to an expression in S' if and only if the semantical rules of S and S' together, without the use of any knowledge about (extra-linguistic) facts, suffice to show that the two expressions have the same extension. Thus, L-equivalence holds, for example, between 'a' in S and 'a′' in S' if we see from the rules of designation for these two individual constants that both stand for the same individual; likewise between 'P' and 'P′', if we see from the rules alone that these predicators apply to the same individuals; between two functors '+' and 'sum', if we see from the rules alone that they assign to the same arguments the same values—in other words, if their full expressions with L-equivalent argument expressions (e.g., '2 + 5' and 'sum(II,V)') are L-equivalent; for two sentences, if we see from the rules alone that they have the same truth-value (e.g., 'Rom ist gross' in

German, and 'Rome is large' in English). Thus, even if the sentences '$2 + 5 > 3$' and 'Gr [sum(II,V),III]' belong to two different systems, we find that they are intensionally isomorphic by establishing the L-equivalence of corresponding signs.

If variables occur, the analysis becomes somewhat more complicated, but the concept of isomorphism can still be defined. We shall not give here exact definitions but merely indicate, with the help of some simple examples, the method to be applied in the definitions of L-equivalence and isomorphism of matrices. Let 'x' be a variable in S which can occur in a universal quantifier '(x)' and also in an abstraction operator '(λx)', and 'u' be a variable in S' which can occur in a universal quantifier 'Πu' and also in an abstraction operator '$\hat{u}$'. If 'x' and 'u' have the same range of values (or, more exactly, of value intensions, § 10), for example, if both are natural number variables (have natural number concepts as value intensions), we shall say that 'x' and 'u' are L-equivalent, and also that '(x)' and 'Πu' are L-equivalent, and that '(λx)' and '$\hat{u}$' are L-equivalent. If two matrices (sentential or other) of degree n are given, one in S and the other in S', we say that they are L-equivalent with respect to a certain correlation between the variables, if corresponding abstraction expressions are L-equivalent predicators. Thus, for example, '$x > y$' in S and '$\mathrm{Gr}(u,v)$' in S' are L-equivalent matrices (with respect to the correlation of 'x' with 'u' and 'y' with 'v') because '$(\lambda xy)\ [x > y]$' and '$\hat{u}\hat{v}[\mathrm{Gr}(u,v)]$' are L-equivalent predicators. Intensional isomorphism of (sentential or other) matrices can then be defined in analogy to that of closed designators, so that it holds if the two matrices are built up in the same way out of corresponding expressions which are either L-equivalent designators or L-equivalent matrices. Thus, for example, the matrices '$x + 5 > y$' and '$\mathrm{Gr}[\mathrm{sum}(u,\mathrm{V}),v]$' are not only L-equivalent but also intensionally isomorphic; and so are the (L-false) sentences '$(x)(y)[x + 5 > y]$' and '$\Pi u \Pi v\ [\mathrm{Gr}[\mathrm{sum}(u,\mathrm{V}),v]]$'.

These considerations suggest the following definition, which is recursive with respect to the construction of compound designator matrices out of simpler ones. It is formulated in general terms with respect to designator matrices; these include closed designators and variables as special cases. The definition presupposes an extended use of the term 'L-equivalent' with respect to variables, matrices, and operators, which has been indicated in the previous examples but not formally defined. The present definition makes no claim to exactness; an exact definition would have to refer to one or two semantical systems whose rules are stated completely.

14-1. ***Definition of intensional isomorphism***

a. Let two designator matrices be given, either in the same or in two different semantical systems, such that neither of them contains another designator matrix as proper part. They are intensionally isomorphic $=_{\mathrm{Df}}$ they are L-equivalent.

b. Let two compound designator matrices be given, each of them consisting of one main submatrix (of the type of a predicator, functor, or connective) and n argument expressions (and possibly auxiliary signs like parentheses, commas, etc.). The two matrices are intensionally isomorphic $=_{\mathrm{Df}}$ (1) the two main submatrices are intensionally isomorphic, and (2) for any m from 1 to n, the mth argument expression within the first matrix is intensionally isomorphic to the mth in the second matrix ('the mth' refers to the order in which the argument expressions occur in the matrix).

c. Let two compound designator matrices be given, each of them consisting of an operator (universal or existential quantifier, abstraction operator, or description operator) and its scope, which is a designator matrix. The two matrices are intensionally isomorphic $=_{\mathrm{Df}}$ (1) the two scopes are intensionally isomorphic with respect to a certain correlation of the variables occurring in them, (2) the two operators are L-equivalent and contain correlated variables.

In accord with our previous discussion of the explicandum, rule b in this definition takes into consideration the order in which argument expressions occur but disregards the place of the main subdesignator. For the intensional structure, in contrast to the merely syntactical structure, only the order of application is essential, not the order and manner of spelling.

§ 15. Applications of the Concept of Intensional Structure

The concept of intensional structure is compared with the concepts of synonymity discussed by Quine and Lewis. The concept is then used for giving an interpretation of belief sentences that seems more adequate than the interpretations discussed earlier (§ 13). Further, the same concept helps in solving the so-called paradox of analysis.

It has often been noticed by logicians that for the explication of certain customary concepts a stronger meaning relation than identity of intension seems to be required. But usually this stronger relation is not defined. It seems that in many of these cases the relation of intensional isomorphism could be used. For example, if we ask for an exact translation of a given statement, say the exact translation of a scientific hypothesis or of the testimony of a witness in court from French into English, we should usually require much more than agreement in the intensions of the sentences, that is, L-equivalence of the sentences. Even if we restrict our attention to designative (cognitive) meaning—leaving aside other meaning components like the emotive and the motivative, although they are often very important even for the translation of theoretical texts—L-equivalence of sentences is not sufficient; it will be required that at least some of the component designators be L-equivalent, in other words, that the intensional structures be alike or at least similar.

Quine explains, without giving a definition, a concept of synonymity which is different from and presumably stronger than L-equivalence. He says: "The notion of synonymity figures implicitly also whenever we use the method of indirect quotations. In indirect quotation we do not insist on a literal repetition of the words of the person quoted, but we insist on a synonymous sentence; we require reproduction of the *meaning*. Such synonymity differs even from logical equivalence; and exactly what it is remains unspecified."[38] We might perhaps think of an explicatum of this concept of synonymity similar to our concept of intensional isomorphism. Quine himself seems to expect that the explication will be found not in semantics but in what we would call pragmatics, because he says that the concept of synonymity "calls for a definition or a criterion in psychological and linguistic terms."

C. I. Lewis[39] gives a definition for the concept of synonymity which shows a striking similarity to our concept of intensional isomorphism, although the two concepts have been developed independently. Since it is interesting to see the points of agreement and of difference, I will quote his explanations at length. "Not every pair of expressions having the same intension would be called synonymous; and there is good reason for this fact. Two expressions are commonly said to be synonymous (or in the case of propositions, equipollent) if they have the same intension, and *that intension is neither zero nor universal*. But to say that two expressions with the same intension have the same meaning, without qualification, would have the anomalous consequence that any two analytic propositions would then be equipollent, and any two self-contradictory propositions would be equipollent." In order to overcome this difficulty, Lewis introduces a new concept: "Two expressions are *equivalent in analytic meaning*, (1) if at least one is elementary [i.e., not complex] and they have the same intension, or (2) if, both being complex, they can be so analyzed into constituents that

[38] [Notes], p. 120.

[39] [Meaning], pp. 245 f. Other concepts used by Lewis will be discussed in the next section.

(*a*) for every constituent distinguished in either, there is a corresponding constituent in the other which has the same intension, (*b*) no constituent distinguished in either has zero intension or universal intension, and (*c*) the order of corresponding constituents is the same in both, or can be made the same without alteration of the intension of either whole expression." As examples, Lewis states that "round excision" and "circular hole" are equivalent in analytic meaning, while "equilateral triangle" and "equiangular triangle" are not, although they have the same intension. He continues: "We shall be in conformity with good usage if we say that two expressions are synonymous or equipollent, (1) if they have the same intension and that intension is neither zero nor universal, or (2) if, their intension being either zero or universal, they are equivalent in analytic meaning."

Thus Lewis' concept of synonymity is very similar to our concept of intensional isomorphism except for one point: He applies this stronger relation only to the two extreme cases of intension, for example, in the field of sentences, only to L-determinate and not to factual sentences. This discrimination seems to me somewhat arbitrary and inadvisable. Let us consider the following examples (in a language which, in distinction to S_1, also contains expressions for finite cardinal numbers and for relations and properties of them):

(i) 'two is an even prime number';
(ii) 'two is between one and three';
(iii) 'the number of books on this table is an even prime number';
(iv) 'the number of books on this table is between one and three'.

The sentences (i) and (ii) have the same intension but are not equivalent in analytic meaning (intensionally isomorphic). The same holds for (iii) and (iv). Now, according to Lewis' definition, (i) and (ii) are not synonymous because they are L-true, analytic; while (iii) and (iv) are synonymous because they are factual, synthetic. It seems to me that it would be more natural to regard (iii) and (iv) also as nonsynonymous, since the difference between them is essentially the same as that between (i) and (ii). The logical operation which leads from (i) to (ii) is the same as that which leads from (iii) to (iv); it is the transformation of '*n* is an even prime number' into '*n* is (a cardinal number) between one and three'.

Now let us go back to the problem of the analysis of belief-sentences, and let us see how the concept of intensional structure can be utilized there. It seems that the sentence 'John believes that D' in *S* can be interpreted by the following semantical sentence:

15-1. 'There is a sentence $\mathfrak{S}_i$ in a semantical system S' such that (*a*) $\mathfrak{S}_i$ in S' is intensionally isomorphic to 'D' in S and (*b*) John is disposed to an affirmative response to $\mathfrak{S}_i$ as a sentence of S'.'

This interpretation may not yet be final, but it represents a better approximation than the interpretations discussed earlier (in § 13). As an example, suppose that John understands only German and that he responds affirmatively to the German sentence 'Die Anzahl der Einwohner von Chicago ist grösser als 3,000,000' but neither to the sentence 'Die Anzahl der Einwohner von Chicago ist grösser als $2^6 \times 3 \times 5^6$' nor to any intensionally isomorphic sentence, because he is not quick enough to realize that the second sentence is L-equivalent to the first. Then our interpretation of belief-sentences, as formulated in 15-1, allows us to assert the sentence 'John believes that the number of inhabitants of Chicago is greater than three million' and to deny the sentence 'John believes that the number of inhabitants of Chicago is greater than $2^6 \times 3 \times 5^6$'. We can do so without contradiction because the two German sentences, and likewise their English translations just used, have different intensional structures. [By the way, this example shows another disadvantage of Lewis' definition of equivalence in analytic meaning. According to part (1) of his definition, the two German sentences are equivalent in analytic meaning if we take '3,000,000' as one sign.] On the other hand, the interpretation of belief-sentences in terms of propositions as objects of beliefs (like (iv) in § 13) would not be adequate in this case, since the two German sentences and the two English sentences all express the same proposition.

An analogous interpretation holds for other sentences containing psychological terms about knowledge, doubt, hope, fear, astonishment, etc., with 'that'-clauses, hence generally about what Russell calls propositional attitudes and Ducasse epistemic attitudes. The problem of the logical analysis of sentences of this kind has been much discussed,[40] but a satisfactory solution has not been found so far. The analysis here proposed is not yet a complete solution, but it may perhaps be regarded as a first step. What remains to be done is, first, a refinement of the analysis in terms of linguistic reactions here given and, further, an analysis in terms of dispositions to nonlinguistic behavior.

[40] Russell, [Inquiry], gives a detailed discussion of the problem in a wider sense, including beliefs not expressed in language; he investigates the problem under both an epistemological and a logical aspect (in our terminology, both a pragmatical and a semantical aspect), not always distinguishing the two clearly. For C. J. Ducasse's conception see his paper "Propositions, Opinions, Sentences, and Facts," *Journal of Philosophy*, XXXVII (1940), 701–11.

The concept of intensional structure may also help in clarifying a puzzling situation that has been called "the paradox of analysis". It was recently stated by G. E. Moore,[41] and then discussed by C. H. Langford,[42] Max Black,[43] and Morton White.[44] Langford[45] states the paradox as follows: "If the verbal expression representing the analysandum has the same meaning as the verbal expression representing the analysans, the analysis states a bare identity and is trivial; but if the two verbal expressions do not have the same meaning, the analysis is incorrect." Consider the following two sentences:

'The concept Brother is identical with the concept Male Sibling.'
'The concept Brother is identical with the concept Brother.'

The first is a sentence conveying fruitful information, although of a logical, not a factual, nature; it states the result of an analysis of the analysandum, the concept Brother. The second sentence, on the other hand, is quite trivial. Now Moore had been puzzled by the following fact: If the first sentence is true, then the second seems to make the same statement as the first (presumably because, if two concepts are identical, then a reference to the one means the same as a reference to the other, and hence the one expression can be replaced by the other); "but it is obvious that these two statements are not the same", he says. Black tries to show that the two sentences do not express the same proposition; he supports this assertion by pointing to the fact that the first sentence, or rather a paraphrasing he gives for it ('the concept Brother is the conjunct of the concept Male and the concept Sibling') refers to a certain nonidentical relation (the triadic relation Conjunct), while the second is a mere identity. White replies that this is not a sufficient reason for the assertion. None of the four authors states his criterion for the identity of "meaning", "statement", or "proposition"; this seems the chief cause for the inconclusiveness of the whole discussion. If we take, as in the terminology used in this book, L-equivalence as the condition for the identity of propositions, then White is certainly right; since the two sentences are L-true and hence L-equivalent to each other, they express the same proposition in our sense. On the other hand, Black feels correctly, like Moore and Langford, that there is an important difference in meaning between the two sentences, because of a difference in meaning between

[41] *The Philosophy of G. E. Moore*, ed. P. Schilpp (1942), pp. 660–67.

[42] "The Notion of Analysis in Moore's Philosophy", *ibid.*, pp. 321–42.

[43] *Mind*, LIII (1944), 263–67 and LIV (1945), 272 f.

[44] *Mind*, LIV (1945), 71 f. and 357–61.

[45] *Op. cit.*, p. 323.

the two expressions for the analysandum ('the concept Brother') and the analysans ('the concept Male Sibling'). The paradox can be solved if we can state exactly what this difference in meaning is and how it is compatible with the identity of meaning in another sense. The solution is quite simple in terms of our concepts: The difference between the two expressions, and, consequently, between the two sentences is a difference in intensional structure, which exists in spite of the identity of intension. Langford saw the point at which the difference lies; he says[46] that the analysans is more articulate than the analysandum, it is a grammatical function of more than one idea; the two expressions are not synonymous but "cognitively equivalent in some appropriate sense". It seems to me that this cognitive equivalence is explicated by our concept of L-equivalence and that the synonymity, which does not hold for these expressions, is explicated by intensional isomorphism.

§ 16. Lewis' Method of Meaning Analysis

Lewis uses, in addition to the concepts of extension and intension which are similar to ours, the concept of comprehension which presupposes the admission of nonactual, possible things. It seems inadvisable to use this conception because it requires a new, more complicated language form. The distinction which Lewis wants to make can better be made with respect to intensions than with respect to things.

I wish to discuss briefly some concepts which have recently been proposed by C. I. Lewis[47] as tools for a semantical meaning analysis. There is a striking similarity between these concepts and our concepts of extension and intension. This similarity is due to the common aim to make some traditional concepts, especially extension and intension, denotation and connotation, more general in their application and, at the same time, more clear and precise.

Lewis explains his chief semantical concepts in the following way: "All terms have meaning in the sense or mode of denotation or extension; and all have meaning in the mode of connotation or intension. The *denotation* of a term is the class of all actual or existent things to which that term correctly applies. . . . The *comprehension* of a term is the classification of all consistently thinkable things to which the term would correctly apply. . . . For example, the comprehension of "square" includes all imaginable as well as all actual squares, but does not include round squares. . . . The *connotation* or *intension* of a term is delimited by any correct definition of it."

It seems that Lewis' concepts of extension and intension correspond closely to our concepts. This is clearly the case for predicators, but perhaps also for sentences and individual expressions. There remains the problem of the necessity and usefulness of Lewis' third concept, that of comprehension. It seems that Lewis follows Meinong[48] in dividing (1) all things (in the widest sense) into impossible or inconceivable things (e.g., round squares) and possible things; and (2) the possible things into actual things (e.g., Plato) and nonactual possible things (e.g., Apollo, unicorns). [Lewis clearly makes the second division. Whether he also makes the first and hence countenances, like Meinong, impossible things is not quite so clear but seems indicated by the formulation that the comprehension "does not include round squares". According to the ordinary conception, in distinction to Meinong's, there are no round squares at all, not even in some particular kind of objects; hence it would be redundant to say that the comprehension "does not include round squares".] Meinong's conception has been critically discussed by Russell[49] and then rejected. Russell's chief reason for the rejection is that the impossible objects violate the principle of contradiction; for example, a round square is both round and nonround, because square. Russell is certainly right in the following respect: Within the logical framework of our ordinary language, we cannot consistently apply the conception of impossible things or even that of possible nonactual things. And, as far as I am aware, neither Meinong nor Lewis nor any other philosopher has constructed or even outlined a language of a new structure which would accommodate those entities. That such a language must be different from the ordinary one is shown by the following example: In the ordinary language we say: 'There are no white ravens and no round squares'. In the new language we would have to say, instead: 'There are white ravens; however, they are not actual, but only possible. And there are round squares; however, they are neither actual nor possible, but impossible.' I have no doubt that a resourceful logician could easily construct a consistent language system of this kind, if we wanted it; he would have to lay down rules for the quantifiers deviating from the ordinary rules in a way suggested by the examples. The

[46] *Op. cit.*, p. 326.

[47] In [Meaning]. This paper is part of a "Symposium on Meaning and Truth", published in four parts in *Philosophy and Phenomenological Research*, Vols. IV (1943–44) and V (1944–45). This symposium also contains a number of other interesting contributions to the development and clarification of semantical concepts. I have elsewhere referred to Tarski's paper [Truth]; I am in close agreement with his conception of the nature of semantics, but he does not discuss the central problems of this book. Concerning these problems, I wish especially to call attention to the papers by C. J. Ducasse (IV, 317–40; V, 320–32) and Charles A. Baylis (V, 80–93).

[48] A. von Meinong, *Untersuchungen zur Gegenstandstheorie und Psychologie* (1904).

[49] [Denoting], pp. 482 f.